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Mass of the charm tetraquark $(c\bar{c}c\bar{c})$ in diquark–antidiquark and di-hadronic states approach

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Abstract The mass of the recently discovered charm tetraquark state has been estimated in the framework of the diquark–antidiquark ($c\bar{c}c\bar{c}$) and in the di-hadronic states approach. In the context of the effective mass approximation, diquarks have been designated as quasiparticles and have been used to study the tetraquark ($c\bar{c}c\bar{c}$) state. It has also been investigated considering it as a di-hadronic state of di- J/Ψ mesons. The results have been compared with the experimental value and other theoretical works. It is observed that the mass of the tetraquark state in the diquark–antidiquark configuration shows very good agreement with the experimental results. The diquark–antidiquark picture in the context of the effective mass approximation seems to describe the ($c\bar{c}c\bar{c}$) system very well.

1 Introduction

Last decade is the years of achievement on the exploration of multiquark states in both the field of theoretical and experimental investigations. In recent years, a number of multiquark states have been discovered in high-energy experiments which cannot be described by usual baryon (qqq) or meson $(q\bar{q})$ configurations. Recently, LHCb [1] has reported an observation of the structure, while studying the mass spectrum of J/Ψ pairs using the proton-proton collision data at center of mass energies of \sqrt{S} = 7.8 and 13 TeV, the mass and the natural width of the narrow X(6900) structure have been measured. They have used two different models to describe structure line shape which give well description of the data obtained. In model I, X (6900) structure is considered as a resonance, whereas the threshold enhancement is described through a superposition of two resonances. The mass and the natural width are determined as $m[X(6900)] = 6905 \pm 11 \ MeV/c^2$ and $\Gamma[X(6900)] = 80 \pm 19 \ MeV$. In model II, the interference between the NRSPS component and a resonance for the threshold enhancement is taken into account, and the mass and the natural width are determined to be $m[X(6900)] = 6886 \pm 11 \ MeV/c^2$ and $\Gamma[X(6900)] = 168 \pm 33 \ MeV$. DO experiment has also observed the existence of the tetraquark state X(5568) [2]. Moreover, LHCb [3] has also announced the discovery of X(4274), X(4500), X(4700) tetraquarks in addition to

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the confirmed existence of Z(4430) state [4]. The BES III experiment and Belle experiment independently were reported on $Z_c(3900)$, the first confirmed four quark states [5]. The existence of the pentaquark charmonium states with the decay of Λ_b^0 was also confirmed by LHCb [6] few years ago, and the intermediate states were identified as $P_c^*(4380)$ and $P_c^*(4450)$. Recently, Aaij et al. (LHCb collaboration) [7] have announced the pentaquark $P_c^+(4312)$ decaying to a proton and a J/Ψ particle. Formerly reported $P_c^*(4450)$ was also observed consisting of two narrow overlapping peaks of $P_c^+(4440)$ and $P_c^+(4457)$.

The tetraquark state T_{4c} was first proposed by Iwasaki [8,9] just after the discovery of J/Ψ meson [10,11]. A number of models and approaches have been proposed by different authors to study the tetraquark states. Jaffe [12] has studied the spectra and dominant decay couplings of multiquark hadrons $Q^2 Q^2$ in the framework of quark bag model. The most important candidates for investigating the tetraquark states are potential model and diquark-antidiquark approach. Debastiani et al. [13] have studied the tetraquark masses both in diquark-antidiquark picture and meson molecule. They have described the diquark using Cornell potential and reproduced the mass of Z(4430). Using the Yukawa potential, they have solved the Schrödinger equation for two-body system describing the charm tetraquark as meson molecule and have obtained the mass of X(3872). W. Chen et al. [14] have estimated doubly hidden charm and bottom masses in diquark-antidiquark configuration for different J^{PC} states and have observed that the masses are higher than the observed spontaneous dissociation threshold of two charmonium mesons while performing the QCD sum rules. H. Chen et al. [15] have also investigated the strong decays of the possible fully charm tetraquarks recently observed by LHCb [1]. Wang et al. [16] have studied the mass spectra of s-wave fully heavy tetraquark states in diquark-antidiquark picture in nonrelativistic quark model with one-gluon exchange coulomb, linear confinement-type potential and hyperfine interaction between the diquark and antidiquark. They have observed that the ground states are 300-450 MeV above the lowest scattering state indicating no bound state in this nonrelativistic scheme. Doubly heavy tetraquark masses have been investigated by a number of authors [17-21] in the context of the constituent quark models and QCD sum rules. The multiquark states like $P_c^*(4380)$ and $P_c^*(4450)$ have been studied by Ghosh et al. [22] in diquark-diquark-antiquark scheme using quasiparticle picture of diquark and also as dihadronic states [24] which have been found to reproduce the observed spectra well. Pal et al. [24] investigated the multiquark states describing the diquarks as composite fermions, whereas Chakrabarti et al. [25] studied the multiquark states with di-hadronic states which also reproduced the masses within the experimental predictions. Excellent reviews on the multiquark states have been done by Chen et al. [26] and Liu et al. [27]. The dynamics of fully heavy tetraquark states are under extensive study. Most of the investigations are model dependent with different results. More experimental observations are needed to understand the full spectra of heavy tetraquark states.

In the current work, we have estimated the mass of the charm tetraquark state in the framework of the diquark–antidiquark formalism in the context of effective mass approximation. Multiquark states represent a new facet of QCD, and their dynamics is both challenging and difficult to understand the context of strong interaction only. Various approaches were made to describe these states. Diquark formalism is one of the most important candidates for multiquark states. During last few years, we have developed a model for describing the diquark as a quasiparticle and its mass is estimated in the framework of effective mass approximation in analogy with the usual condensed matter physics [28]. In another approach, the mass of the tetraquark state has also been studied as di-hadronic state using a van der Waals' type of interaction between the hadrons.

2 Formalism

2.1 Estimation of the tetraquark mass in the diquark-antidiquark configuration

Two quarks are assumed to be correlated to form a low energy configuration designated as diquark within a hadron. We have described a diquark as quasiparticle in effective mass approximation in analogy to the concept of quasiparticle in condensed matter physics [29, 30]. Diquark is supposed to behave like a quasiparticle in an analogy with an electron in the crystal lattice which behaves as a quasiparticle [31]. An electron in a crystal is subjected to two types of forces: One is the effect of the crystal field ($-\nabla V$), and the other is external force (**F**) which accelerates the electron. Under the influence of these two forces, it behaves like a quasiparticle having velocity v whose effective mass m^* reflects the inertia of electron in a crystal field and can be represented as,

$$m^* \frac{dv}{dt} = \mathbf{F},\tag{1}$$

The bare electron (with normal mass m) is affected by the lattice force $-\nabla V$ and the external force F so that

$$m\frac{dv}{dt} = \mathbf{F} - \frac{d\mathbf{V}}{dx}.$$
(2)

From (1) and (2), the ratio of the normal mass (m) to the effective mass (m^*) can be represented as,

$$m/m^* = 1 - \frac{1}{\mathbf{F}} \left[\frac{d\mathbf{V}}{dx} \right]. \tag{3}$$

The difference between the effective and normal mass is attributed to the lattice force.

An elementary particle in vacuum may be suggested to be in a situation exactly resembling that of an electron in a crystal. It is well known that a quasiparticle is a low-lying excited state whose motion gets modified by the interactions within the system. The strong interaction is characterized by two features like confinement and asymptotic freedom. We have assumed that under the combined force of confinement and asymptotic freedom, a diquark in hadron behaves like a quasiparticle and its mass gets modified simulating the many-body interaction in a hadron [32]. We have suggested that the potential $V = -\frac{2}{3} \frac{\alpha_s}{r}$ where α_s is the strong coupling constant and this potential resembles the crystal field on a crystal electron, so that the average force $\mathbf{F} = -a\mathbf{r}$ resembles the external force where 'a' is a suitable constant. The potential can be represented as,

$$V_{ij} = -\frac{\alpha}{r} + (F_i \cdot F_j) \left(-\frac{1}{2}Kr^2\right),\tag{4}$$

where the coupling constant $\alpha = (2/3)\alpha_s$, $F_i \cdot F_j = -(2/3)$, and K is the strength parameter. Hence, V_{ij} may be represented as,

$$V_{ij} = -\frac{(2/3)\alpha_s}{r} + ar^2,$$
 (5)

where a = K/3.

The ratio of the constituent mass and the effective mass of the diquark (m_D) can be expressed using equation (3), and we have obtained,

$$\frac{m_q + m_{q'}}{m_D} = 1 + \frac{\alpha_s}{3ar^3}.$$
 (6)

 $m_q + m_{q'}$ represents the normal constituent mass of the diquark, m_D is the effective mass of the diquark, and 'r' is the radius parameter of diquark. With $\alpha_s = 0.2$ [33], a = 0.06

*GeV*³ [34], $r_{cc}(\text{scalar})= 0.576 \text{ fm}$ [35], $r_{cc}(\text{vector})= 0.579 \text{ fm}$ [35], $m_c = 1.71 \text{ GeV}$ [36], we have estimated the effective masses of the diquarks in the framework of the quasiparticle model [32] that is $m_{[cc]_0} = 3.2696 \text{ GeV}$ for scalar diquark and $m_{[cc]_1} = 3.2705 \text{ GeV}$ for vector diquark.

We have assumed that the mass of the antidiquark can be estimated using the same formalism as the diquark. It may be mentioned that when a collective excitation occurs in a complicated microscopic system, the formulation is equally applicable for the electrons and the holes. The diquark and the antidiquark masses of same flavor possess the same effective mass in the current approach.

The mass formula for the tetraquark state with the relevant diquark–antidiquark configuration is simply additive with binding energy and spin interaction between them which runs as,

$$M = m_D + m_{\overline{D}} + E_{BE} + E_S,\tag{7}$$

where m_D , $m_{\overline{D}}$ are diquark and antidiquark masses, respectively, E_{BE} is the binding energy of the diquarks, and E_S is the spin term. The binding energy of the diquarks can be described as an interaction acting between them and can be expressed in the form of potential as,

$$E_{BE} = (\Psi(r_{12})|V(r)|\Psi(r_{12})).$$
(8)

where $\Psi(r_{12})$ is the wave function of the tetraquark state and r_{12} is the radius parameter of the tetraquark . V(r) is assumed to be linear or harmonic type of potential between diquark and antidiquark. The potentials are expressed as

$$V(r) = a'r, (9)$$

$$V(r) = a'' r^2.$$
 (10)

To estimate E_{BE} , we have used the wave functions for the ground state of the tetraquark from the statistical model [37,38]

$$|\Psi(r_{12})|^2 = \frac{315}{64\pi r_{12}^{\frac{9}{2}}} (r_{12} - r)^{\frac{3}{2}} \theta(r_{12} - r), \qquad (11)$$

$$|\Psi(r_{12})|^2 = \frac{8}{\pi^2 r_{12}^6} (r_{12}^2 - r^2)^{\frac{3}{2}} \theta(r_{12} - r), \qquad (12)$$

corresponding to the linear type background potential and harmonic type of background potential, respectively. $\theta(r_{12} - r)$ is the usual step function and $r_{12} = r_1 + r_2$, where r_1 and r_2 represent the individual radius of the diquark and antidiquark constituting the tetraquark, respectively.

The spin term is expressed as [39],

$$E_S = \frac{8}{9} \frac{\alpha_S}{m_D m_{\overline{D}}} \overrightarrow{S_1} \cdot \overrightarrow{S_2} |\Psi(0)|^2.$$
(13)

where $\overrightarrow{S_1} \cdot \overrightarrow{S_2}$ is the spin interaction of the corresponding states. We have taken the values of $a' = 0.06 \ GeV^3$ [33] and $a'' = 0.11 \ GeV^2$ [40] for linear type and harmonic type of potential. The masses of the $(c\bar{c}c\bar{c})$ tetraquark state for different combinations of scalar and vector diquarks have been estimated using the relation (7) and are reported in Table 1.

2.2 Estimation of the tetraquark mass in the di-hadronic state approach

In a different approach, the tetraquark state has been described as di-hadronic molecule consisting of two mesons assuming a van der Waals' type of molecular interaction acting between them [25,41,42]. Treating the tetraquark as meson–meson system, the mass formula for the low-lying di-hadronic molecule runs as:

$$M_{Total} = M_1 + M_2 + E_{BE} + E_{SD}, \tag{14}$$

where M_1 , M_2 represent the masses of the constituent hadrons, respectively, E_{BE} represents the binding energy of the di-hadronic system, and E_{SD} represents the spin-dependent term.

The binding energy has been already expressed in equation (8); here, r_{12} is the radius parameter of the di-hadronic molecule and V(r) is the di-hadronic molecular potential which can be expressed as [25,41,42],

$$V(r) = \frac{-k_{mol}}{r} e^{-C^2 r^2/2},$$
(15)

where k_{mol} is the residual strength of the strong interaction molecular coupling and C is the effective color screening of the confined gluons. It may be mentioned that the residual interaction of the confined gluon is considered similar to the van der Waals' interaction and assumed to be due to asymptotic expression $(r_{12} \rightarrow \infty)$ of the residual confined one-gluon exchange interaction with strength k_{mol} [25,41,42]. $\Psi(r)$ is the wave function of the dihadronic state. To estimate E_{BE} , we have used the wave function for the ground state of the hadronic molecule from statistical model expressed in equation (11), corresponding to the linear type of background potential. r_{12} is the radius of the hadronic molecule, $r_{12} = r_1 + r_2$, where r_1 and r_2 represent the individual radii of the hadrons constituting the molecule, respectively, and $\theta(r_{12} - r)$ is the usual step function. Using equations (8), (11) and (15), we get E_{BE} as,

$$E_{BE} = \frac{2.25k_{mol}}{r_{12}} {}_2F_2[(1, 1.5), (2.25, 2.75), -\beta],$$
(16)

where $_2F_2$ is the relevant hypergeometric function, $\beta = C^2 r_{12}^2/2$, C = 50 MeV [43], and $k_{mol} = 0.59$ [25]. The radius of $c\bar{c}$ has been used as $r(J/\psi) = 2.005 \ GeV^{-1}$ [44] and the radius of $c\bar{c}$ as $r(\eta_c) = 1.845 \ GeV^{-1}$ [45].

The spin hyperfine interaction can be expressed as [46],

$$E_{SD} = \frac{8}{9} \frac{\alpha_s}{M_1 M_2} \overrightarrow{S_1} \cdot \overrightarrow{S_2} |\Psi(0)|^2.$$
(17)

 α_s is the strong interaction constant, S_1 and S_2 are the spins of the hadrons involved, and $|\Psi(0)|^2$ is the di-hadronic wave function at the origin. E_{SD} has been estimated subsequently using the relation (17) with $\alpha_s = 0.2$ [33]. The mass of X(6900) has been estimated using equation (14) with the mass of the respective hadrons (M_1) and (M_2) [47] and is displayed in Table 2.

3 Conclusions and discussion

In the current work, we have estimated the mass of the recently discovered charm tetraquark system in the diquark–antidiquark framework. Tetraquark as di-hadronic state $(c\bar{c})$ $(c\bar{c})$ has also been studied. The results are displayed in Tables 1 and 2, respectively. The diquark–antidiquark approach of investigating the tetraquark mass shows a very good agreement

State	Quark content	Experimental <i>mass(GeV)</i>	Our work (<i>GeV</i>) with linear potential	Our work (<i>GeV</i>) with harmonic potential	Other theoretical works (GeV)
X(6900)	$[cc]_0[\overline{cc}]_0$		6.8847	7.2852	6.797 [4 9], 6.477 [5 2]
	$[cc]_1[\overline{cc}]_0$	6.905 ± 0.011	6.8865	7.2901	6.899 [<mark>26</mark>]
	$[cc]_1[\overline{cc}]_1$		6.8884	7.2950	6.956 [<mark>49</mark>]
	[00]][00]]		0.0001	/12/00	0.7

Table 1 Estimated tetraquark masses of X(6900) with $[cc][\overline{cc}]$ configuration using diquark-antidiquark approach

Table 2 Estimated tetraquark masses of X(6900) with $[c\overline{c}][c\overline{c}]$ configuration using di-hadronic approach

Particle	Spin	Experimental mass (GeV)	Our work (GeV)	Other theoretical works (<i>GeV</i>)
$(\eta_c \eta_c)$	0		6.1546	5.969 [<mark>50</mark>]
$(\eta_c J/\psi)$	1	6.905 ± 0.011	6.2634	6.020 [<mark>50</mark>]
$(J/\psi J/\psi)$	2		6.3736	6.1154 [<mark>50</mark>]

with the experimental value. We have considered three types of diquark-antidiquark configurations, i.e., i) scalar diquark-scalar antidiquark $[cc]_0[cc]_0$, ii) vector diquark-scalar antidiquark $[cc]_0[\overline{cc}]_1$, iii) vector diquark-vector antidiquark $[cc]_1[\overline{cc}]_1$. The scalar and vector diquarks (antidiquarks) differ for their radii. We have estimated the binding energies considering the wave functions corresponding to the linear and harmonic oscillator type of potential. The splitting between scalar-scalar and vector-scalar diquark-antidiquark configurations is found to be 0.0018 GeV and that of scalar-scalar and vector-vector configuration is 0.0037 GeV in linear potential model and 0.0049 GeV and 0.0098 GeV in harmonic oscillator potential model. It may be mentioned that Liu et al. [48] have estimated the mass as 6.518 GeV in potential motivated models. They have observed the mass splitting between the two configurations as 0.031 GeV. Our results also compare favorably with the theoretical work of Wu et al. [49]. They have estimated the charm tetraquark mass in the framework of color magnetic interactions and predicted the value as 6.797 GeV. Chen et al. [14] have studied heavy charm and bottom tetraquarks using QCD sum rules with different interpolating currents and estimated the mass of 0⁺⁺ state as 6.44 GeV to 6.82 GeV, whereas Debastiani et al [50] have estimated the mass of the 0^{++} state as 5.9694 GeV. They have found out the mass of the diquark as 3.1334 GeV and pointed out that the spin-dependent interaction has substantial contribution in diquark-antidiquark formalism. We have obtained the scalar diquark mass as 3.2696 GeV and 3.2705 GeV for vector diquark in the framework of the quasiparticle model of diquark in effective mass approximation.

We have also studied the charm tetraquark as two charm meson states such as $(\eta_c \eta_c)$, $(\eta_c J/\psi)$ and $(J/\psi J/\psi)$ with suitable potential acting between them. The results we have obtained in the framework of di-hadronic states are found to yield less values compared to effective mass approximation but well inside the range of the experimental mass value. Debastiani et al. [13] have investigated charm tetraquark as meson molecules, and values are found to be smaller than the experimental predictions. Our result is also in agreement with the results of Karlinar et al. [51] and Lloyd et al. [52].

We have explored the charm tetraquark mass X(6900) in both the quasiparticle model and di-hadronic states approach. The current investigation shows that the quasiparticle picture of diquarks reproduces the tetraquark mass which is in excellent agreement with the experimental value. However, it may be mentioned that the most uncertain parameter used in our model is the radius of different diquarks which have been given as input from the current knowledge. We have estimated the percentage variation of tetraquark masses due to small variation of radius parameter. It has been calculated here that for a variation of ± 0.2 GeV^{-1} in radius, the percentage error in tetraquark masses varies from 2.17% to 2.36% for diquark-antidiquark configuration, whereas for di-hadronic states, the variation in masses is from 2.21% to 3.30% approximately. However, the estimation of the tetraquark mass in the di-hadronic concept is found to be less than the experimental value. But the quasiparticle model of diquark-antidiquark configuration gives encouraging results. It takes into account the effect of background through the effective mass approximation. This model is also found to be successful in describing the baryons as diquark-quark states and a number of lightheavy exotic states [53–55]. It also yields very good results for tetra heavy quark states. It may be stated that the quasiparticle model describes the multiquark states very well and may not be far from reality. We will study the higher states of the tetraquarks in both the charm and bottom sector in our future work.

Data Availability Statement This manuscript has associated data in a data repository. [Authors' comment: All data included in this manuscript are available upon request by contacting with the corresponding author.]

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